A unifying physical theory of eddy covariance

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Summary

We show that the conventional theory guiding eddy covariance (EC) measurements contains two flaws. First, it neglects the close coupling between mass and total (internal, kinetic, and potential) energy transfers in turbulent flows on the Earth's surface, an open thermodynamic system. Second, it inadequately constrains offset errors in vertical wind velocity measurements. These two flaws lead to underestimation of the magnitude of diurnal Earth-atmosphere exchanges of sensible heat while the second flaw also leads to biases in the measured exchanges of gases. We form a physical theory of EC from the fundamental equations of coupled mass and energy transfer derived from the first principles of physical fluid mechanics and thermodynamics. Contrary to the conventional theory, the physical theory simultaneously conserves mass and total energy. New approaches to constraining wind velocity offset errors are also proposed. We demonstrate the improvements brought about by the physical theory at contrasting EC sites. EC measurements around the world should be conducted according to the new theory while past measurements should be corrected. Our development of the physical EC theory removes a major uncertainty in Earth system research.

1. What is the objective of the physical theory of eddy covariance?

- To develop a non-intrusive approach to measuring:

$$\overline{NEE}_{\mathrm{g}} = \int_0^{z_m} \bar{s}_{\mathrm{g}} dz + \bar{J}_{\mathrm{gz}}\Big|_{z=0} \qquad \overline{NEE}_{\mathrm{SH}} = \int_0^{z_m} \bar{q}_{\mathrm{c}} dz + \bar{G}_{\mathrm{z}}\Big|_{z=0}$$

NEE_{g} :Net ecosystem exchange (NEE) rate of a gas species g

s_{g} :Net production rate of the gas species per unit volume

$J_{\mathrm{gz}}\big|_{z=0}$: Molecular diffusional flux of the gas from ground

NEE_{SH} :NEE of sensible heat

q_{c} :Net rate of heating of air per unit volume

$G_{\mathrm{z}}\big|_{z=0}$:Conductive heat transfer from ground

2. Overcoming theoretical barriers

- Apply the first principles of physical fluid mechanics and thermodynamics
- Focus on NEE, rather than flux
- Think about a controlled air volume, rather than a plane
- Realize in an open thermodynamic system, energy exchange:
  - is always coupled with mass exchange
  - Can occur in the forms of internal, kinetic, and potential energies
- Avoid untested assumptions
- When assumptions must be made, the uncertainties caused by the assumptions are quantified

3. Overcoming practical barriers

- The EC approach requires exceedingly precise absolute accuracy in the measured mean vertical wind velocity $\bar{w}$
- Sonic anemometers have offset errors ~ 2 cm s^{-1} in $\bar{w}$
- Direct use of sonic $\bar{w}$ under typical atmospheric conditions can lead to an error in CO_2 exchange over 320 μmol m^{-2} s^{-1}
- Conventional approach to constraining $\bar{w}$ requires the application of ideal gas law which is not valid in turbulent flows due to non-equilibrium gas states
- The physical theory constrains $\bar{w}$ by using either the exchange ratio of coupled gases (*e.g.*, CO_2 and water vapor) or surface energy balance

4. Derivations of the fundamental equations

- Mass exchange: start from the mass conservation equation in the Eulerian reference frame (Gu *et al*. 2012, 2013)

$$\frac{\partial \rho_{\mathrm{g}}}{\partial t} + \nabla \cdot (\boldsymbol{u}\rho_{\mathrm{g}}) = -\nabla \cdot \boldsymbol{J}_{\mathrm{g}} + s_{\mathrm{g}}$$

- Heat exchange: start from the total energy conservation equation in the Lagrangian reference frame (newly derived)

$$(\rho_{\mathrm{d}} c_{\mathrm{vd}} + \rho_{\mathrm{v}} c_{\mathrm{vv}})\frac{dT}{dt} + \frac{1}{2}(\rho_{\mathrm{d}} m_{\mathrm{d}} + \rho_{\mathrm{v}} m_{\mathrm{v}})\frac{d(\boldsymbol{u}\cdot\boldsymbol{u})}{dt} + (\rho_{\mathrm{d}} m_{\mathrm{d}} + \rho_{\mathrm{v}} m_{\mathrm{v}}) g\frac{dz}{dt} = q_c - \nabla\cdot\boldsymbol{G} + \epsilon_{\mathrm{i}} - e_{\mathrm{k}} - \nabla\cdot(p\boldsymbol{u})$$

- Why total energy? Heating air can result in exchanges of internal, kinetic and potential energies (IE, KE and PE)
- Why Lagrangian reference frame? It is too difficult to write down the total energy conservation equation in the Eulerian reference frame

5. The final equations for applications at typical EC flux sites

$$\overline{NEE}_{\mathrm{TE}} = \overline{NEE}_{\mathrm{IE}} + \overline{NEE}_{\mathrm{KE}} + \overline{NEE}_{\mathrm{PE}}$$

$$\overline{NEE}_{\mathrm{SH}} = \overline{NEE}_{\mathrm{TE}} - \overline{NEE}_{\mathrm{TEd}} - \overline{NEE}_{\mathrm{TEv}}$$

$$\overline{NEE}_{\mathrm{IE}} = \bar{S}_{\mathrm{IE}} + \bar{F}_{\mathrm{ENEC}} + \bar{F}_{\mathrm{ENMM}}$$

$$\overline{NEE}_{\mathrm{KE}} = \bar{S}_{\mathrm{KE}} + \bar{F}_{\mathrm{KEEC}} + \bar{F}_{\mathrm{KEMM}} + \bar{C}_{\mathrm{KE}}$$

$$\overline{NEE}_{\mathrm{PE}} = \bar{S}_{\mathrm{PE}} + \bar{F}_{\mathrm{PEEC}} + \bar{F}_{\mathrm{PEMM}}$$

$$\overline{NEE}_{\mathrm{TEd}} \cong c_{\mathrm{vd}}\overline{NEE}_{\mathrm{d}}\bar{T}_{\mathrm{s}} \qquad \overline{NEE}_{\mathrm{TEv}} \cong c_{\mathrm{vv}}\overline{NEE}_{\mathrm{v}}\bar{T}_{\mathrm{s}}$$

$$\overline{NEE}_{\mathrm{d}} = \bar{S}_{\mathrm{d}} + \bar{F}_{\mathrm{dEC}} + \bar{F}_{\mathrm{dMM}} \qquad \overline{NEE}_{\mathrm{v}} = \bar{S}_{\mathrm{v}} + \bar{F}_{\mathrm{vEC}} + \bar{F}_{\mathrm{vMM}}$$

$$\bar{F}_{\mathrm{EC}} = \overline{w'x'} \qquad \bar{F}_{\mathrm{MM}} = \bar{w}\bar{x}$$

Interested in the details of the theory or in applying it at your flux site? Contact Lianhong-gu@ornl.gov

Gu et al (2012) The fundamental equation of eddy covariance and its application in flux measurements. Agricultural and Forest Meteorology 152, 135-148.

Gu L (2013) An eddy covariance theory of using O_2 to CO_2 exchange ratio to constrain measurements of net ecosystem exchange of any gas species. Agricultural and Forest Meteorology 176, 104-110.

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